Concentration in a Population Model Structured in Age and Phenotypic Trait

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Résumé

We study a mathematical model of a biological population structured by age and phenotypical trait. The interactions between individuals with a phenotypic variability usually lead to competition and selection of the fittest individuals. The goals of this work are to describe the long time behaviour of the solution to a renewal type equation, and then to derive properties that illustrate the selection phenomena.
We consider the following model: [see pdf]
with x the age, and y a phenotypical trait. The unknown m is the population density and the function A(x, y) represents the rate at which the population ages with the trait y. The parameter is used for a time rescaling. The quantity ρ is the total density of the population. Here the mortality effect features a saturation term ρ (t) and a death rate d(x, y) $>$ 0. The condition at the boundary x = 0 describes the birth of newborns that happens with rate b(x, y) $>$ 0 and with the probability kernel of mutation M .
As a first step, we begin with a simpler model by only considering the competition and do not take into account the mutations. The analysis is lead by the study of an eigenvalue problem, with eigenelements depending on the structuring variables of the model. Then we tackle the problem with mutations, which leads to the study of a constrained Hamilton-Jacobi equation, following earlier works on similar issues, that poses technical difficulties.
This is a joint work with Vincent Calvez (ENS Lyon), Samuel Nordmann (EHESS) and Benoit Perthame (UPMC).

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